

On the Algebraic Definition of Programming Languages

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Software crisis

Complex systems, many bugs.

How do we improve **existing software**?

Develop tools to

- ▶ test code (Attila Kovács)
- ▶ analyse code (Melinda Tóth, István Bozó)
- ▶ refactor code in a correct way (Dániel Horpácsi)

Software crisis

Complex systems, many bugs.

How do we improve **future software**?

Develop new programming languages

- ▶ it should be easy to write correct code
- ▶ hard to write incorrect code

Why do we need different programming languages?

Programming languages are the same:

- ▶ They are just different ways to express CPU instructions.
- ▶ They are all Turing-complete.

Why do we need different programming languages?

Programming languages are the same:

- ▶ They are just different ways to express CPU instructions.
- ▶ They are all Turing-complete.

They are not the same:

- ▶ If the language suits the problem, we can express the solution **directly**.
- ▶ If not, we need to **encode** the solution.

Programming is the process of encoding

Domain specific languages:

- ▶ HTML for hypertext
- ▶ P4 for programming switches (Sándor Laki)
- ▶ Erlang for programming mobile networks, distributed systems
- ▶ Alan Kay's example:
 - ▶ operating system with GUI, web browser, spreadsheet, word processor
 - ▶ all in 20thousand lines of code

Difficulties of creating new languages

- ▶ Design the language such that it is sound
 - ▶ Prove its properties
- ▶ Implementation, lots of technical details
 - ▶ Improvements recently, e.g. LLVM
- ▶ Interface to other languages

People usually don't bother: instead, they follow the hammer principle.

What is a domain specific language for describing languages?

This is our research project.

What is a language?

A language is a collection of programs.

What is a program?

What is a program?




(1) number

What is a program?

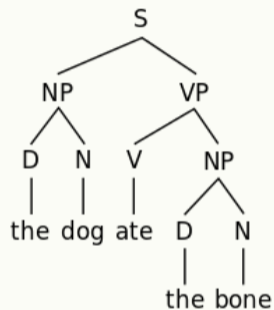
```
ApplicationInstanceManager.setApplicationInstanceListener(new ApplicationInstanceListener() {  
    @Override  
    public void newInstanceCreated(String[] args) {  
        System.out.println("New instance detected, args length=" + args.length);  
        if (args.length > 1)  
            if (args[0].equals("-open")) {  
                if (facKinView.getFileChooser().isVisible())  
                    facKinView.getFileChooser().cancelSelection();  
                facKinView.openFile(args[1]);  
                //TODO: nem szabadna, hogy ilyen kelljen:  
                //waiting 1 second  
                try { Thread.sleep(1000); } catch (InterruptedException ex) {}  
                //long t0, t1;t0 = System.currentTimeMillis(); do { t1 = System.currentTimeMillis();  
                for (int i = 2; i < args.length; i++) {  
                    facKinView.openFile(args[i]);  
                }  
            }  
    }  
}
```

(2) string

decode  encode

(1) number

What is a program?



(3) syntax tree

parsing $\left(\begin{array}{c} \curvearrowright \\ \curvearrowleft \end{array} \right)$ print

(2) string

decode $\left(\begin{array}{c} \curvearrowright \\ \curvearrowleft \end{array} \right)$ encode

(1) number

What is a program?

(4) syntax tree with bindings

scope checking $\left(\begin{smallmatrix} \uparrow \\ \downarrow \end{smallmatrix}\right)$ pick variable names

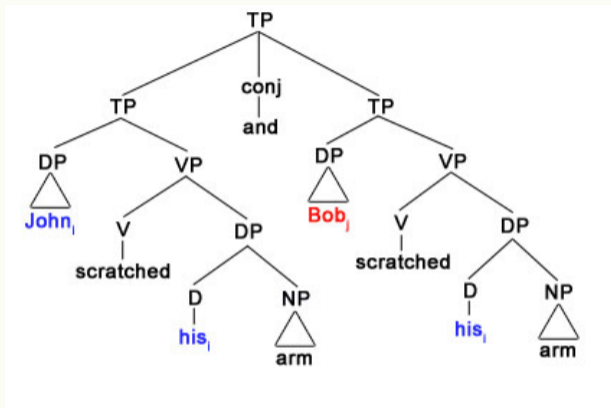
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parsing $\left(\begin{smallmatrix} \uparrow \\ \downarrow \end{smallmatrix}\right)$ print

(2) string

decode $\left(\begin{smallmatrix} \uparrow \\ \downarrow \end{smallmatrix}\right)$ encode

(1) number



What is a program?

(5) well typed syntax

type checking $\left(\begin{array}{l} \uparrow \\ \downarrow \end{array} \right)$

(4) syntax tree with bindings

scope checking $\left(\begin{array}{l} \uparrow \\ \downarrow \end{array} \right)$ pick variable names

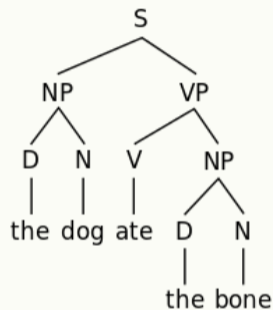
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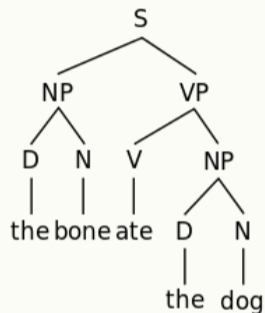
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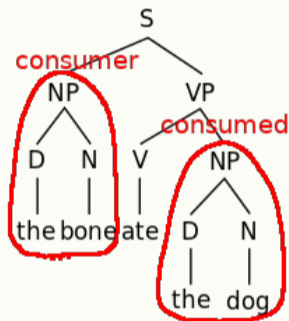
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(2) string

decode $\left(\begin{array}{c} \uparrow \\ \downarrow \end{array}\right)$ encode

(1) number



What is a program?

(5) well typed syntax

type checking $\left(\begin{smallmatrix} \nearrow \\ \searrow \end{smallmatrix}\right)$

(4) syntax tree with bindings

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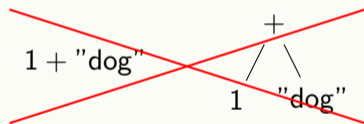
(3) syntax tree

parsing $\left(\begin{smallmatrix} \nearrow \\ \searrow \end{smallmatrix}\right)$ print

(2) string

decode $\left(\begin{smallmatrix} \nearrow \\ \searrow \end{smallmatrix}\right)$ encode

(1) number



What is a program?

(6) well typed syntax quotiented

normalise

$$1 + 2 = 3$$

(5) well typed syntax

type checking

$$\begin{array}{c} + \\ / \quad \backslash \\ 1 \quad 2 \end{array} = 3$$

(4) syntax tree with bindings

scope checking pick variable names

(3) syntax tree

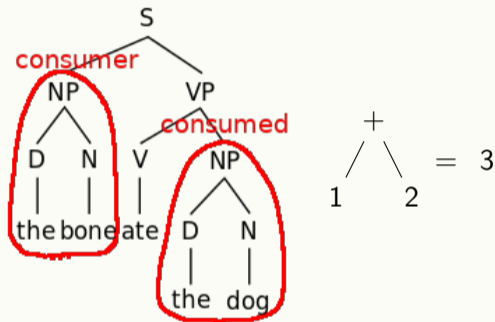
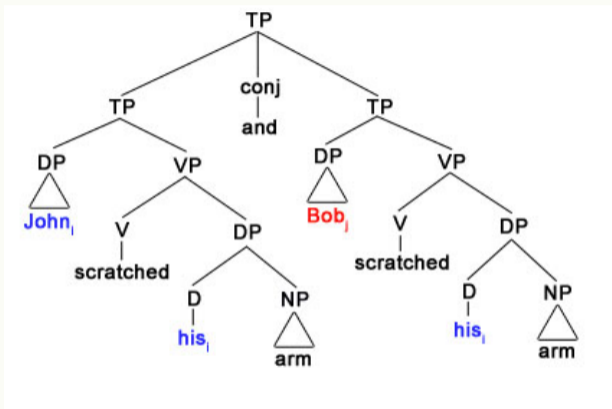
parsing print

(2) string

decode encode

(1) number

A language is a collection of:



Mathematicians call this a **generalised algebraic structure**, programmers call it a **quotient inductive-inductive type**.

Results

- ▶ We created the first programming language which supports quotient inductive-inductive types (QIIT).
 - ▶ It is based on Martin-Löf's type theory and we call it setoid type theory.
 - ▶ Syntax (as a QIIT) and constructive semantics.
 - ▶ A prototype implementation in Haskell.
- ▶ We developed the metatheory of QIITs.
 - ▶ A concrete description.
 - ▶ We proved their consistency, constructive semantics.
- ▶ We developed techniques to formalise metatheoretic results in existing languages based on type theory.

Further goals

- ▶ Theoretical: show decidability of type checking for setoid type theory
- ▶ Theoretical: reduction of QIITs to basic combinators
- ▶ Practical: industrial strength implementation
- ▶ Practical: implement existing programming languages as QIITs
- ▶ Long term: implement setoid type theory in itself



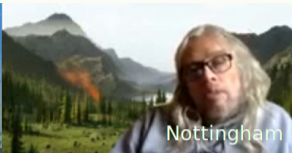
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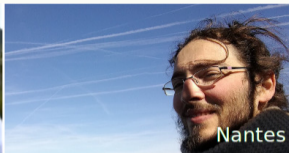
Team and collaborators



Budapest



Nottingham



Nantes



Budapest



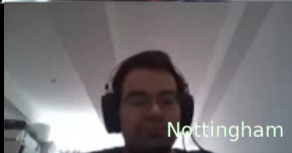
Karlsruhe



Darmstadt



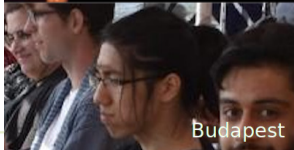
Budapest



Nottingham



Sidney



Budapest



Budapest



Gothenburg

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Summary

- ▶ A good programming language admits direct representations of the domain
- ▶ Programming languages are usually **encoded** instead of **directly** represented
- ▶ Our hypothesis: the reason for this is that there is no programming language supporting QIITs
- ▶ We are developing such a language: the theoretical foundations are mostly done
- ▶ We would like to develop a practical implementation



Q&A



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